

UNIT 1: ELEMENT OF PHILOSOPHY OF MATHEMATICS AND PROOF IN MATHEMATICS.

INTRODUCTION:

Philosophy of mathematics is that branch of philosophy which attempts to answer question such as “why is mathematics useful in describing nature,” “In which sense, if any, do mathematics entities such as number exist” and “why and how are mathematical statements made? The philosophy of mathematics, as a discipline has dealt for many centuries with the issue of what is the nature of mathematics. This age-old discussion is far from being conclusive, rather it is evolving as each thinker contributes his or her view of looking at the different facets which mathematics present as a discipline. This philosophical debate is indispensable since teaching and learning mathematics is influenced by the perspectives adopted, and it is because mathematics has had a central role in the advancement of societies that defining its nature, role and methodology has become a central, ideological and cultural issue

In everyday life, we frequently reach conclusions based on anecdotal evidence. This habit also guides our work in the more abstract realm of mathematics, but mathematics requires us to adopt a greater level of skepticism. Examples- no matter how many-are never a proof of a claim that covers an infinite number of instances.

OBJECTIVES

By the end of this unit, you should be able to:

1. distinguish between philosophy of mathematics and mathematics education.
2. differentiate between logicism, formalist and intuitionism schools.
3. discuss the influence of philosophy of mathematics on mathematics pedagogy
4. explain what a proof is
5. give reasons why we prove
6. describe some proof methods

HOW TO STUDY THIS UNIT

1. Read the unit carefully in order to understand the differences between philosophy of mathematics and philosophy of mathematics education
2. Study the key words and make sure you can distinguish between them.
3. Note carefully the divergent views about mathematics education

WORD STUDY

Philosophy; Logicism; Formalists; Intuitionist; Behaviorism; Constructivism.

PHILOSOPHY OF MATHEMATICS EDUCATION

Mathematics beliefs can be studied in the light of major philosophies and pedagogical stances on the nature, teaching and learning of mathematics. The philosophical and pedagogical stances portray well-structured representations that have been sometimes the result of hundreds of years of collective reflection. These macro stances are useful given their capacity to articulate a background from which other relatively minor issues can be discussed. On the other hand, each individual holds his or her own conception of mathematics teaching and learning.

These conceptions are unique in that they are the results of their own formal or informal contemplation of reality. Both macro and micro conceptions of mathematics are significant because they represent human beliefs that influence instructional behaviour.

The Philosophy of Mathematics

Early attempts to develop a methodological foundation of mathematics attempted to vindicate it as a discipline free of error that did justice to its arrogant and secular epithets as “the most perfect of all sciences (Lakatos, 1986, p31), the “mother’ (Mura, 1995, p.390)”, the queen of all sciences (McGinnis, Randy, Shaman, McDttie, Hurtles, King, & Watanebic, 1996, p17)” a science in its own right” (Mura, 1995, p.33). Others began to doubt the dogmatic assumption that mathematics was actually an ‘a priori infallible’ enterprise, whose methodology could be perfectly delineated and whose development was amenable to being formulated through a formal and universal system.

An alternative conception therefore began to evolve in which mathematics was concerned as a fallible, empirical or quasi-empirical discipline.

In the last century, the nature of mathematics became a central issue for educationalists as it had been before for the philosophers. A personal philosophy of mathematics education ascertains the way we learn and teach mathematics within the classroom and the school environment (Soutwell, 1999). If mathematics is, as the Platonist tradition suggested, just an entity out there waiting to be discovered, then it will be enough for schools to present the curriculum instruction as a mere collection of facts, definition and algorithms. In that regard, teaching mathematics would be like just transmitting an immutable body of knowledge that students have to accept as a perennial fact without any reasoning. However, if mathematics is an empirical activity, then learners are in the position of constructing their own mathematics knowledge regardless, of how different their methodology may be from the cannon of orthodox and classical mathematics.

We owe the first attempts to secure an error-proof methodology of mathematics to the Ancient Greek. It was Euclid (365-275 B.c) who dared to explain mathematical reasoning through a consistent network of postulates, corollaries, axioms and theorems. For nearly two millennia, the academic community used Euclid’s reasoning model to advance mathematical knowledge. However, it was mainly Labatchevsky (1793-1856) who dethroned Euclid’s infallibility by proving that the fifth of Euclid’s five postulates was not absolutely true (Bald on 1984). Subsequent developments in mathematics showed that conventional methods of mathematical proof led to the paradoxes and therefore the search for an alternative infallible

method became central at the beginning of this century. Consequently, three paradigms were advanced to secure the foundation of mathematics, namely logicism, symbolism and intuitionism and they become known as the foundationalist movement.

Logicism

Logicism is basically a form of platform realism in which mathematics is seen as a set of abstract realms that exist externally to human creation. According to logicians; all mathematical concepts can be reduced to abstract properties that can be derived through logical principle. Logicism has been criticized because of its failure to enunciate an unerring system of mathematical truth; its discouraging appropriate discussion of basic mathematical concepts such as plane, line sets and so on. Logicism has also been criticized for its obsession for strict logical reasoning learning, little room for intuition and conjecture which many see as powerful generators of creative thinking (Goodman, 1986).

Formalism (Symbolism)

Formalists share the logicist view that logic is necessary. However they argue that mathematical knowledge is brought about through the manipulation of symbols that operate by prescribed rules and formulas and whose understanding should be accepted a priori. Formalism has been criticized because it has little space left for creative thinking, the unfeasibility of creating an inclusive mathematical system due to the need for a concomitant extensive list of definitions, properties, rules and the like, and the deifying of the mastery of mathematical symbolism over meaningful inference and intuition.

Intuitionism

In the intuitionist faction, mathematics is conceived as an intellectual activity in which mathematical concepts are seen as mental constructions regulated by natural laws. These constructions are regarded as abstract objects that do not necessarily depend on proofs. Brouwer, the founder of intuitionism, rejects the classical stance of categorizing proofs as either true or false and instead argues that other possibilities for claiming mathematical induction comes before and it is independent of logic. Likewise, intuition and imagination are seen as early and necessary psychological stages in the process of invention. The main critics to intuitionism argue that mathematical constructions are not only mentally but also socially constructed. These critics also argue that absolute freedom of thought is detrimental to mathematics. It has also been said that intuitionists' biggest downfall lies in enunciating their theory using formalist methods (Goodman, 1986)

The crisis and failing of the three factions described above in securing mathematics as an abstract, absolutist, universal and infallible system was followed by an increasing interest in exploring mathematics as an activity which was practically fallible, situated and socially and personally constructed. The movement was labeled "quasiempirical" because it proposed that mathematics did not actually belong to the category of hard sciences such as physics in which something out there is to be discovered. Instead, mathematics is human creation born of and nurtured from practical experience, always growing and changing, open to revision and challenges, and whose claims of truth depend on guessing by speculation and criticism, by

the logic of proofs and refutations (Lakatos, 1976 p5). According to Polya (1986), mathematics is both demonstration and creation. Demonstration is achieved by proofs while creation consists of plausible reasoning that includes guessing. Mathematical methods therefore are not perfect and cannot claim absolute truth. Mathematical truth is not absolute but relative because in fact truth is time dependent (Grabiner, 1886) and space dependent (Wilder, 1986). Time dependent because what is scientifically true today might be a falsehood in the future as theoretical assumptions change, as occurred with the theories of Euclid and Ptolemy. Mathematical methods are also space dependent because different peoples and different cultures have different ways of doing and validating their mathematical knowledge (Ascher, 1991).

The transition from the foundationalist approach, with its emphasis on pure mathematics, to the quasi-empirical approach was followed by a renewed interest in the application of mathematics, as seen above from the foundationalists realm of mathematics, a fact that took them away from an emphasis on application of mathematics (Robitaille & Dirks, 1982, J.Rogerson, 1989). If pure mathematics is to have any value by itself, it cannot be attained by sacrificing the value of engaging in the application of mathematics.

Foundationalists' overvaluing of pure mathematics neglected the fact that the origin and goal of mathematics was the search for solutions to humanity's proximal environment. In fact, one of the merits of Euclid's geometry is that he designed his deductive method from empirical evidence (Baldor 1984). Mathematics therefore had grown parallel to and serving the so-called hard sciences and it is to this practical and interactive experience to which mathematics owes most of its greatness. (Putnam, 1986). For Putnam (1986) the greatness of mathematics did not reside only in its ability to go beyond the realm of concrete entities, nor in the beauty of their proofs, but in its concomitant power in providing utilitarian solutions to the bewildered homo sapiens in their settlement on earth.

Influence of the philosophy of mathematics on the pedagogy of Mathematics

The formalist and logicist paradigms as Hersh (1979) and Rogerson (1994) have argued have had a strong influence on mathematics education in this century and therefore students have learned what mathematics is. The new mathematics wave, set theory, the emphasis on mutation, symbolism, functions and relations, more stress on analytical rather than descriptive geometry, and behavioural perspectives on education have certainly been part of the foundationalist legacy which influenced the school mathematics curriculum and models of teacher education in the world (Laurenson, 1995; Moreira & Noss, 1993, Robitaille & Dirks 1982, Thom, 1986)

As the second half of the last century continued to evolve, the international mathematics education community was keener to consider and adopt the quasi-empirical conception of mathematics, no matter how eclectic this view was. Major reform documents such as the U.S Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, NCTM, 1989), Professional Standards for the Teaching of Mathematics (NCTM, 1991), Assessment Standards for School Mathematics (NCTM, 1995), Principles and Standards for School Mathematics (NCTM, 2000), the U.K Cockerft Report (Cockcroft,

1982) the National Statement On Mathematics for Australian Schools (Australian Education Council 1991), the Statement of Principle for Mathematics K-12 and The Nature of Mathematics Teaching and Learning (Board of studies. New south Wala, 1996) were inspired in different degrees by the principle of “Knowing mathematics is doing mathematics” (NCTM, 1989 p?) thus reflecting the quasi –empirical approach.

The quasi- empirical approach parallels in many respects the main tenets of the socio-constructivist theory; although it is worthy to note that while the former constitutes a philosophical view on the nature of mathematics the latter focuses its attention on the psychological underpinnings of teaching and learning mathematics.

For many years there has been a debate in education on the advantages and disadvantages of socio-constructivism and behaviorism. These two philosophies on teaching and learning mathematics can be depicted as two contrasting views and both have influenced the way mathematics is being taught in schools (Marland, 1994).

Socio- constructivism or constructivism in shorter terms, as opposed to behaviorism models of teaching and learning claim that knowledge should not be transferred from one individual to another in educational environments. For constructivist educationalist, knowledge must be actively constructed as the learner is an entity with previous experiences that must consider as a “knowing being”. Learning is therefore seen as an adaptive and experiential process rather than a knowledge transference activity (Candy, 1991). As learners encounter new situation they look for similarities and differences against their own cognitive schemata. These contrasts, also called cognitive perturbations, are the end-product of confliction knowledge waiting to be resolved through reorganizing schemes of knowledge (Phillip, 1995).

In constructivist terms, learning depends on the way each individual learner looks at a particular situation and draws his/her own conclusions. People therefore determine their own knowledge based on their own way of processing information and according to his/her own beliefs and attitudes towards learning (Bigg & Moore, 1993). Constructivism therefore gives recognition and value instructional strategies in which students are able to learn mathematics by personally and socially constructing knowledge. Constructivist learning strategies include more reflective oriented learning activities in mathematics education such as exploratory and generative learning. More specifically, these activities include problem solving-group learning, discussion and situated learning (Murphm, 1997 wood cobb, and Yackal, 1991).

Behaviorism

Behaviorism focuses on the manipulation of external conditions to the learner to modify behaviours that eventually lead to learning. In a behaviorism-oriented environment completion of tasks is seen as ideal learning behaviour and mastering basic skills require students to move from basic tasks to more advanced tasks. In addition, learning is considered a function of rewarding and reinforcing. Likewise the emphasis is on correct answers rather than of partially correct answers (Elleot, Kratochwill, and Trawlers, 1996). Inspired by linear programming theories developed particularly during the Second World War, learning and teaching in behaviorist terms is a matter of optimizing and manipulating the instructional environment towards the fulfillment of rigidly and specifically designed educational objectives.

In addition, behaviorists saw the student’s affective domain as different from the cognitive domain. The Bloom Taxonomy, for example, classifies educational objectives in cognitive, affective and psychomotor domains (Krathwohi, Bloom, and Masia 1964). They categorise emotions “as imaginary constructs” that are causes of behaviour (McLeod, 1992). Consequently behaviourism assumes that certain emotions and attitudes can influence behaviour although, in general, affective issues are neglected (McLeod, 1992). Teachers and students’ minds were seen as “black-boxes” or machines (Shavelson Stern, 1981) in which attitudes and behaviour occur somehow or are even not relevant (Newspor, 1978).

It has been said that behaviorism emphasises a process- product and teacher- centredness model of instructions that have been prevalent in classroom teaching and in teacher education programmes in the twentieth century (Marland, 1994).

A behaviorist teaching style in mathematics education tends to rely on practices that emphasize rote learning and memorization of formulas, one-way to solve problems, and adherence to procedures and drill. Repetition is seen as one of the greatest means to skill acquisition. Teaching is therefore a matter of means to reach those objectives and situated learning is given little value in instruction (Leder, 1994). This over emphasis on procedure and formulas resembles traditional formalist and logicist ideas.

It is worthy to add that while most of the literature on mathematics education centre around the dialogue between the constructivist and the behaviorist movements, it is apparent that their differences have been described by educationalists in reform documents under other educational forms. These terms basically discriminate between the teaching of specific facts and a type of instruction that fosters independence. It should be noted that like any other theoretical model, these representations are over simplification of reality and therefore many educational variables are excluded.

The list below shows the different terms used in this discussion

Figure: *Divergement Views In Mathematics Education*

Behaviorist Perspective	Constructivist Perspective	Source
Behaviorism	Constructive	Candy (1991)
Traditional	Progressive	O’Laughm& Campbell
Mimetic	Transformational	Jackson (1986)
Basic Skills	Higher order thinking	Schmidt & Kennedy (1990)
Content	Process	Schmidt& Kennedy (1990)
Positivist	Relativist	Laurenson (1995)
Subject- centred	Child –centred	Sosniak Ethington, & Vardas (1991)

Behaviorist Perspective	Constructivist Perspective	Source
Transmission and factual procedural knowledge	Emphasis on qualitative transformations in the outlook of the learner	Sosniak etal, 1991
Euclidean	Quasi-empirical	Lerman (1983)
Absolutist	Fallibility	Lerman (1983)
Technical- Positivism	Constructivism	Taylor (1990)
Traditional	Nontraditional	Raymond (1997)
Transmission	Child- centeredness	Perry Howard, & Tracey
Transmission	Constructivist	Nisbet & Warien (2000)

ACTIVITY 1

Do the following exercises

- 1 What are the foundationalist schools’ thought as related to mathematics education?
- 2 Distinguish between constructivist and behaviorist view about mathematics learning. Illustrate with examples.

WHAT IS A PROOF

A **proof** is a logical argument that establishes the truth of a statement. The argument derives its conclusions from the premises of the statement, definitions, and ultimately, the postulates of the mathematics system in which the claim is based. By **logical**, we mean that each step in the argument is justified by earlier steps. That is, that all of the premises of each deduction are already established or given. In practice, proofs, may involve diagrams that clarify, words that narrate and explain, symbolic statements, or even a computer program. The level of detail in a proof varies with the author and the audience. Many proofs leave out calculations or explanations that are considered obvious, manageable for the reader to supply, or which are cut to save space or to make the main thread of a proof more readable. In other words, often the overarching objective is the presentation of a convincing narrative.

Postulates are a necessary part of mathematics. We cannot prove any statement if we do not have a starting point. Since we base each claim on other claims, we need a property, stated as a postulate that we agree to leave unproven. The absence of such starting points would force us into an endless circle of justifications. Similarly, we need to accept certain terms (e.g, “point” or “set) as undefined in order to avoid circularity. In general however, proofs use justifications many steps removed from the postulates.

Before the nineteenth century, postulates (or **axioms**) were accepted as true but regarded as self-evidently so. Mathematicians tried to choose statements that seemed irrefutably true—an obvious consequence of our physical world or number system. Now, when mathematics create new axiomatic systems, they are more concerned that their choices be interesting (in terms of the mathematics to which they lead), logically independent (not redundant or derivable from one another), and internally consistent (theorems which can be proven from the postulates do not contradict each other).

Why Do We Prove?

i To Establish a Fact with Certainty

There are many possible motives for trying to prove a conjecture. The most basic one is to find out if what one thinks is true is actually true. Students are used to us asking them to prove claims that we already know to be true. When students investigate their own research question, their efforts do not come with a similar guarantee. Their conjecture may not be true or the methods needed may not be accessible. However, the only way that can be sure that their conjecture is valid, that they have in fact solved a problem, is to come up with a proof. Mathematical truth do tend to stand the test of time. When students create a proof themselves, they are less likely to think of the result as ephemeral. A proof convinces the prover herself more effectively than it might if generated by someone else.

ii. To Gain Understanding

“I would be grateful if anyone who has understood demonstration would explain it to me.”- Fields Medal winner Pierre Deligne, regarding a theorem that he proved using methods that did not provide insight into the question.

There are proofs that simply prove and those that also illuminate. As in the case of the Deligne quote above, certain proofs may leave one unclear about why a result is true but still confident that it is. Proofs with some explanatory value tend to be more satisfying and appealing. Beyond our interest in understanding a given problem, our work on a proof may produce techniques and understandings that we can apply to broader question. Even if a proof of a theorem already exists, an alternative proof may reveal new relationships between mathematical ideas. Thus proof is not just a source of validation, but an essential technique in mathematics.

iii. To Communicate Ideas to Others

Often, mathematicians (of both the student and adult variety) have a strong conviction that a conjecture is true. Their belief may stem from an informal explanation or some convincing cases. They do not harbor any internal doubt, but there is a broader audience that retains some skepticism. A proof allows the mathematician to convince others of the correctness of their idea.

iv. **For the Challenge**

Difficult tasks can be enjoyable. Many mathematical problems are not of profound significance, yet their resolution provides the person who solves them with considerable gratification. Such success can provide a boost in self-esteem and mathematical confidence. The process of surmounting hurdles to a proof can have all of the thrill of a good mystery. Student (and adults) is justifiably excited when they solve a problem unlike any they have previously encountered and which no one else may have ever unraveled.

v. **To Create Something Beautiful**

The more students engage in mathematics research, the more they develop their own aesthetic for mathematical problems and methods. The development of a proof that possesses elegance, surprises us, or provides new insight is a creative act. It is rewarding to work hard to make a discovery or develop a proof that is appealing.

vi **To Construct a Language Mathematical Theory**

We rarely consider mathematical ideas in a vacuum. Our desire to advance to broader mathematical problems is often a source of motivation when we attempt a proof. For example, a number of mathematicians spent many years attempting to characterize a class of objects known as simple group (Horgan). Their cumulative efforts resulted in thousands of pages proofs that together accomplished the task. Many of these proofs, significant in their own right, were of even greater value because of their contribution to the larger understanding that the mathematics community sought.

vii **Proof methods**

The list of proof techniques is endless. Providing students with a repertoire of a few powerful general methods can give them tools that they need to get started proving their conjectures. These first techniques also whet students' appetites to learn more. When students begin work within a new mathematical domain, they will need to learn about the tools (representation, techniques, powerful theorems) common to the problems that they are studying.

Some Proof Methods Are:

- Proof by induction
- Proof by contradiction
- The pigeonhole principle
- Parity in proof
- Invariants

Example as Disproof and Proof

An example cannot prove an affirmative statement about an infinite class of objects. However, a single example, called a **counterexample**, is sufficient to disprove a conjecture and prove the alternative possibility. For example, we know of many even perfect numbers (Weisstein). The discovery of single odd perfect number would be an important proof that such numbers, conjectured not to exist, are possible.

Proof by Exhaustion

When a conjecture involves a finite set of objects, we can prove the conjecture true by showing that it is true for every one of those objects. This exhaustive analysis is sometimes the only known means for answering a question. It may not be elegant, but it can get the job done if the number of instance to test is not overwhelmingly large.

Proof Pending a Lemma

One of the more exciting experiences in mathematics is the recognition that two ideas are connected and that the truth of one is dependent on the truth of the other. Often a student will be working on a proof and discover that they have a line of reasoning that will work if some other claim is true. Encourage the student to develop their main argument and then return to see if they can fill in the missing link. A claim that is not a focus of your interest, but which you need for a larger proof, is called a **lemma**. As students working on a common problem share their discoveries through oral and written reports, they may recognize that a fellow researcher has already proven a needed lemma. Alternatively, they may realize that their conjecture is a straightforward consequence of a general result that another classmate has proven. We call a theorem that readily follows from an important result a **corollary**. These events contribute enormously to students' understanding of mathematics as a communal activity.

There are many well-known cases of theorems that mathematicians have proven pending some other results. Of course, that means that they are not actually theorems until **the** lemma has been established. What is a theorem in these situations is the connection between two unproven results. For example, Gerhard Frey proved that if a long-standing problem known as the Taniyama- Shimura conjecture were true, Fermat's Last Theorem (MacTutor) must be as well. This connection inspired Andrew Wiles to look for a proof of the Taniyama- Shimura conjecture.

WHEN IS A PROOF FINISHED?

How do we know that we have proven our conjecture? For starters, we should check the logic of each claim in our proof. Are the premises already established? Do we use the conclusions to support a later claim? Do we have a rigorous demonstration that we have covered all cases?

We next need to consider our audience. Is our writing clear for someone else to understand it? Have we taken any details for granted that our reader might need clarified? Ultimately, the acceptance of a proof is a social process. Do our mathematical peers agree that we have a

successful proof? Although we may be confident in our work, unless others agree, no one will build upon or disseminate our proof. Our theorem may even be right while our proof is not. Only when our peers review our reasoning can we be assured that it is clear and does not suffer from logical gaps or flaws. If a proof is unclear, mathematical colleagues may not accept it. Their clarifying questions can help us improve our explanations and repair any errors. On the other, mathematical truth is not democratically determined. We have seen many classes unanimously agree that a false assertion was true because the students failed to test cases that yielded counterexample. Likewise, there have been classes with one voice of reason trying to convince an entire class of non-believers. The validity of a proof is determined over time - readers need time to think, ask questions, and judge the thoroughness of an exposition. Students should expect to put their proofs through the peer review process.

When do peers accept a proof? When they have understood it, tested its claims, and found no logical error. When there are no intuitive reasons for doubting the result and it does not contradict any established theorems. When time has passed and no counterexample has emerged. When the author is regarded as capable (“I don’t understand this, but Marge is really good at math”). Some of these reasons are more important than others, but all have a role practice.

How to End a Proof

Since one reason we tackle proofs is for the challenge, we are entitled to a modest “celebration” when a proof is completed. The nicest honour is to name a theorem after the student or students who prove it. If you dub proofs after their creator (e.g, Laura’s Lemma or the Esme- Reinhard Rhombus Theorem Pythagoras theorem) and have them posted with their titles, students will be justifiably proud. Give conjectures title, as well, in order to highlight their importance and as a way to promote them so that others will try to work on a proof.

Introduce students to the traditional celebration: ending a proof with “Q.E.D.” Q.E.D. is an acronym for “quod erat demonstrandum,” Latin for “that which was to be demonstrated.” At the end of a proof by contradiction, students can use “Q.E.A.,” which stands for “quad est absurdum” and means, “ that which is absurd” or “we have a contradiction here.” These endings are the understated mathematical versions of “TaDa!” or Eureka!” Modern, informal equivalents include AWD (“and we’re done”) and W⁵ (“which was what we wanted”) (Zeitz, p 45). We have also seen “☺” and “MATH is PHAT!” at the end of students’ proofs. Professional publications are now more likely to end a proof with a rectangle (□) or to indent the proof to distinguish it from the rest of a discussion, but these are no fun at all.

ACTIVITY

1. Why is proof important in mathematics?
2. Distinguish between Lemma and Corollary
3. Give at least 7 proof methods
4. How do we end a proof?
5. Proof that the sum of the angle on a straight is two right angle.

SUMMARY

- From this unit you have learnt the differences between philosophy of mathematics and philosophy of mathematics education.
- You also have learnt about foundationalist movement in mathematics education.
- You have been introduced to the constructivists and behaviorists thought on mathematics teaching and learning
- The unit also treated about the divergent views on mathematics education
- The meaning of proof in mathematics has been explained.
- The importance of proofing was discussed.
- The methods of proof were also explained.

ASSIGNMENT

- 1 Trace the factors that lead to the quasi-empirical movement in mathematics education

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UNIT 2: HISTORY OF MATHEMATICS TEACHING IN NIGERIA AND THE PHILOSOPHY OF CURRENT MATHEMATICS CURRICULA

INTRODUCTION

Components of mathematics teacher education are various and relevant either from mathematical point of view or in pedagogical sense. As several studies have pointed out, “the history of mathematics can play a valuable role in mathematics teaching and learning. The use of history in mathematics education links psychological learning processes with historical and epistemological issues and this link can be ensured by epistemology.

In considering the features of the interaction between history of mathematics and educational practice, a wide range of view and experiences can be examined. Different levels can be considered with reference to teaching and learning processes. A first one is related to anecdotes presentation and it can be useful in order to strengthen pupils’ conviction

OBJECTIVES

By the end of this unit, you should be able to

- (i) state the need for history of mathematics
- (ii) trace the history of mathematics education in Nigeria to the present day
- (iii) state the objectives of mathematics teaching in both primary and secondary schools in Nigeria
- (iv) relate mathematics teaching to the overall objectives of national philosophy of education.

HOW TO STUDY THIS UNIT

1. Read the unit carefully in order to understand how the various stages in the history of mathematics education are connected.
2. Get a copy of national policy of education by your side when reading the unit.

HISTORY OF MATHEMATICS EDUCATION IN NIGERIA

A historical account of mathematics education and mathematics curriculum development in Nigeria is sprinkled in the publications of mathematics education and researches (Fajemidagba, (1991). According to Fajemidagba 1991), the formal teaching of mathematics started with arithmetic, a component of mathematics at the primary and post primary schools. Arithmetic was compulsory for every primary school student and must be passed before a student could obtain the “Primary School Leaving Certificate”. The same condition holds for the Teacher Training Colleges –either Grade III or Grade II. At the secondary school level mathematics was taught in compartments –algebra, arithmetic and geometry with trigonometry. The schools were more guided by the examination syllabi of the Cambridge

local syndicate, the West African Examinations Councils (WAEC) or the University of London General Certificate of Education Ordinary or Advanced Level. Later in 1968 the WAEC published three alternative syllabi for mathematics at the Ordinary level-alternatives A, B and C. Alternative A contained arithmetic process, algebra, absolute geometry and trigonometry. Alternatives B contained the same items as in A but with additional topic in coordinate geometry. These alternatives A B and C were for the average students and mathematically capable students in mathematics. The syllabus spanned content areas in pure mathematics (elementary calculus) mechanics and statistics. At the Advanced level there were syllabi for each, for pure mathematics, applied mathematics, pure with applied mathematics and mathematics with physics. These examination syllabi were invariably turned into teaching syllabi by mathematics teachers.

The major cause of the changes in school mathematics curricula and programmes was the launching of the Sputnik, the first earth satellite in space, in November 1957 by the Russians. The event according to Griffiths and Howson (1974) had an enormous effect on American complacency about their superiority in engineering capability over that of the Russians. The result of the doubt were a series of hot debates and arguments on the suitability or otherwise of the school mathematics and curricula. These occurrences were called “issues and forces” behind the reformation of school mathematics curricula contents, which also affect various reforms in school mathematics programme in Nigeria. This was the effect of the “revision syndrome” which swept across the world (Fajemidagba, 1991). Elementary mathematics was introduced to primary schools in order to replace the arithmetic in the teacher training college. The attempt was to produce primary school teachers who could teach the elementary mathematics. In service programmes were organized for serving teachers for the purpose of retaining them. As a continuation of the innovation, entrance examinations into secondary schools, which used to contain arithmetic and English Language were changed to mathematics and general knowledge, which includes some mathematics concepts and English Language. The National Common Entrance Examination took a lead in the changes. Moreover, the trend forced nearly all primary schools in Nigeria to commence the teaching of mathematics to their pupils.

The introduction of “Modern Mathematics” to secondary school pupils marked the beginning of the teaching of mathematics as a composite or integrated body of knowledge, rather than the former compartmentalization into arithmetic, algebra and geometry with trigonometry that were separately taught to pupils. The introduction of modern mathematics was the aftermath of the wave of changes in schools mathematics that traversed the industrialized nations. As reported by Onuche (1978) the outcomes of the **inter-national** conference on science in the advancement of New States in Retrovoth, Israel in July 1960 contained some recommendations for innovations in school mathematics for African States. The recommendations led to the establishment of the African Mathematics Programme (A.M.P.) known as the Entebbe Mathematics, because the programme was housed in Entebbe, a city in Uganda.

Moreover, the programme was called African Mathematics Programme because its initiators had in mind mathematical contents which would center on the needs of Africans and which would correlate with African cultural values and traditions without any prejudices against modern education.

The Entebbe mathematics started in 1962 as reported by Osibodu (1988). There were three macro goals of the project:

- (a) to prepare and publish instructional materials in mathematics for use in schools, teachers' colleges and inservice institutions in African States,
- (b) to trial – test the developed materials in schools and teachers' colleges and
- (c) to train teachers for the proper use of the developed instructional materials. Thus, the goals seem to parallel the Research, Development and Dissemination (R.D. & D.) paradigm. That is, those involved in the project were assigned to “fish” out local materials that would meet the mathematical needs of African children, through research and development of what were discovered to be available to Africans and what might constitute the needs of the African child.

Further investigation revealed that Mr. Hugh P. Bradely was the director of the A.M.P. and the Education Development Center, Newton, Massachusetts housed its administration. This was the case because the U.S. government provided the lion share of the fund for the programme. African and American mathematicians were employed at various workshops to develop mathematics courses and materials that would go into the programme. The mathematicians were engaged as consultants. Some of them lectured at various institutes organized for the training of African mathematics teachers on how to use the mathematics materials. Eleven training institutes were held in Ethiopia, Ghana, Liberia, Malawi, Nigeria, Uganda, Sierra Leone and Tanzania. Moreover, a four-week meeting was scheduled for the development of textbooks. Two textbooks were produced for use in schools – Advanced Mathematics 1, and Additional Mathematics I and II. These books were adopted for mathematics teaching and learning in African countries that accepted the African Mathematics Programme. In addition to the secondary school textbooks, a series of six textbooks were developed for primary schools. The mathematics concepts treated in the books include the structure of Arithmetic, foundations of geometry, measurement, functions and probability, the number line and fractions.

As discerned by Ohuche (1978), Nigeria participated actively in every facet of the African Mathematics Programme. However, the only successfully implemented project under the auspices of the A.M.P. was the Lagos State Primary School Mathematics experiment. The project started in January 1964 under the directorship of Professor Alele-Williams. Other states in Nigeria did not accept the modernization of mathematics curriculum, dubbed, as “modern mathematics” at the primary school level. According to Williams (1974) the Lagos State Ministry of Education accepted the introduction of innovations into primary school mathematics curriculum. Thus, the original Entebbe primary mathematics textbooks were adopted for use in Lagos State Primary Schools.

As a part of the project, primary school teachers in Lagos State were retrained through inservice activities. This was in tune with the objectives of the African Mathematics Programme. Also, the execution, that is, the actual use of the mathematics materials did not commence until 1971. However, traditional textbooks (e.g. the Larcombe Arithmetic Series) were allowed to compete with the Entebbe Primary mathematics series. Furthermore, many teachers did not understand the objectives and the subject-matter content of the series. This

was a reflection of the inadequacies in the mathematical training received by primary school teachers. The majority of the primary school teachers studied arithmetic per se in the course of their training. Hence, they could not be described as “specialists” in mathematics teaching at the primary school level. It is of interest to note that the funding agencies were the United State Agency for International Development U.S.A.I.D. and the Ford Foundation.

In 1970, the West African Regional Mathematics Programme (W.A.R.M.P) was formed by West African English speaking countries in order to cater for the mathematical needs of the Anglophone West African Countries. It was an off-shoot of the A.M.P.

Unfortunately, Nigeria declined to participate in the programme (see Ohuche, 1978). Rather Nigeria established the Nigerian Educational Research Council (now Nigeria Educational Research and Development Council (N.E.R.D.C.) in 1969. This body was given the responsibilities of charting a new course in the modernization of school mathematics curriculum in Nigeria. Before dealing with the activities of the N.E.R.D.C., a terse historical account of the Joint School Project will be given. The textbooks developed from the project were recommended for the teaching of “modern mathematics” in Nigeria.

Another dimension to the modernization of the mathematics curriculum was the introduction of Joint School Project (J.S.P.) textbooks into Nigeria. The J.S.P. was originated in February, 1964 by eight teachers from three secondary schools in Ghana. The eight teachers worked as a sub-committee of the Mathematical Association of Ghana. The initial work on the project commenced at Achimota School, Ghana. As revealed, the organizational agencies were the University of Ghana and the Ministry of Education, Accra, Ghana. However, the funding agencies were Nuffield Foundation. The Center for Curriculum Renewal and Education Development Overseas, London, the Guinness Award Scheme and the World Confederation of Organization of the Teaching Profession. The macro goals of the project were:

- (1) to produce a “new mathematics” course for secondary schools in course on practical work to make up for a lack of opportunity for such experience in students cultural and educational background,
- (2) making the course relevant to the environments and future needs of students, and to encourage learning by understanding rather than rote,
- (3) to give an intuitive development of the course, since experience did show that over emphasis on logic kills all interest and hinders progress and
- (4) to modernize mathematics topics in order to achieve the objectives above.

The J.S.P. course was at two levels: basic, for those students who will do no mathematics after the Ordinary level, and special, for those who will specialize in science subjects after the Ordinary level. Further, the project differed from similar projects because emphasis was placed on how mathematics arises naturally in the environment and how it can be applied in various situations rather than placing emphasis on the logical structure of mathematics itself. The originators of the project felt that this approach would prove to be much more valuable for the average student in Ghana. A set of textbooks were produced by the originators of the project for both students and teachers. Several secondary schools in Nigeria adopted the textbooks for mathematics teaching and learning, and the W.A.E.C. started examining

students on a syllabus based on the project in June, 1969 (for further technical details on J.S.P. see Lockard 1970). This writer could not identify how the J.S.P. textbooks were introduced to Nigerian secondary school students.

Modern mathematics programmes however, did not succeed in Nigeria. A catalogue of criticisms were levied against the programmes. The contents of the modern mathematics curriculum were meant for potential mathematicians not for consumers of mathematical ideas who form the bulk of the students' population. Also, parents found the contents difficult to perceive and understand and thus could not help their children. Finally, parents and teachers pointed up that the textbooks are too wordy for lower secondary school students. It is very difficult to really pin down a logical argument against the modernization of mathematics in Nigeria. It is well understood and accepted that there was a small number of mathematics teachers at the secondary school level and a majority of the teachers were not adequately trained to teach the new mathematics. Also, it could be conjectured that a majority of the mathematics teachers' training lacked indepth study of mathematics content; the amount of mathematics learned at the various training Colleges might not be adequate for the indepth understanding of the content of the "new" mathematics curriculum. The unification of mathematical ideas has been a serious issue for mathematics educators and mathematicians. Attempts are being made to postulate a unifying theme for mathematics at the primary and secondary school levels. The new mathematics introduced either through the Entebbe project or the J.S.P. project seems to have used set-theory as the "unifying concept". Other researchers in mathematics have also proposed "function" as unifying theme for secondary school mathematics. By and large, no unified mathematical idea has been provided for students. Undoubtedly, researchers in mathematics education and mathematics continue to work on the formulation of a unifying theme for mathematics teaching in schools.

The controversy surrounding the modern mathematics prompted the Federal Government of Nigeria to organize a conference, which was held in Benin City in December, 1977. Mathematics educators and mathematicians were invited as participants at the conference. However, it was at the conference that the ban on modern mathematics was announced by the Government. The announcement was a shock to many participants because that was contrary to their expectations.

After the announcement, the Nigerian Education Research and Development Council (N.E.R.D.C.) was mandated to "work out" appropriate mathematics curricula for the Nigerian children. It was stipulated that the curricula should take cognizance of the needs of the nation and those of the children. Sequel to that, a workshop was organized at the University of Ibadan in February, 1977. The workshop's focus was on the study of the problems facing the teaching of mathematics in Nigerian schools and colleges. It was at the workshop that proposals were made for the development of a set of new mathematics curriculum for schools and colleges in Nigeria. The set of mathematics curricula was to adequately meet the provisions contained in the New National Policy on Education. In the policy, it is stipulated that a 6-3-3-4 system of education will be embarked upon in Nigeria. In the system, mathematics is a core subject for primary school pupils, junior and senior secondary school students.

Several workshops were organized by the N.E.R.D.C. in order to produce set of mathematics curricula that meets the requirements of the 6-3-3-4 system of education. A mathematics curriculum was developed for primary schools, one for the junior secondary schools and two – a general mathematics and Further Mathematics – curricula for the senior secondary schools. The Further Mathematics curriculum is meant for those who have the intent of studying mathematics beyond the senior secondary school level or those who intend to study mathematics related disciplines, e.g. Physics, Chemistry, Engineering, Architecture and the likes. Another mathematics curriculum, was developed for the Teachers' Training Colleges.

These mathematics curricula are in operation in every school and teachers' grade II colleges in Nigeria. Attempts are being made to produce text-books based on the curricula.

The N.E.R.D.C. and other publishers have produced a set of six text-books for the primary schools, covering primary one through primary six. Also the Mathematical Association of Nigeria, (M.A.N.) has produced mathematics text-book for junior secondary year one through year three. Other Government agencies and publishing houses are producing mathematics text-books based on the new set of mathematics curricula. However, no serious evaluation has been performed on the curricula and the accompanying mathematics text-books. This is a challenge to mathematics educators and mathematicians.

ACTIVITY

1. What were the major segments into which mathematics was divided in the Primary School?
2. Discuss the role Sputnik play in the development of mathematics education in Nigeria.
3. What are the factors that led to the failure of modern mathematics curriculum in Nigeria?
4. Examine the objectives of mathematics teaching in the primary and the secondary schools and discuss how they can help in realizing the Nigerian national objectives.

SUMMARY

You have learnt in this unit

- that Arithmetic was the major focus in the pre-independence mathematics curriculum in the primary school.
- That the launching of the Sputnik by then USSR in the fifties led to world wide reform in mathematics education.
- About the objectives of mathematics teaching in both primary and secondary school in Nigeria.

ASSINGMENT

1. How can mathematics teaching in the primary schools be used to achieve some of the national objectives as stated in the national policy of education?

REFERENCE

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UNIT 3: COGNITIVE THEORIES OF LEARNING AND MATHEMATICS TEACHING

INTRODUCTION

There are various theories underlying the teaching of Mathematics. These theories give us the various paradigms upon which classroom practices are based.

This unit introduces you to the various theories and the methods for mathematics teaching.

Some of the learning difficulties in mathematics are also discussed.

OBJECTIONS:

At the end of this unit, you should be able to:

- (1) discuss the cognitive theory of learning
- (2) explain the theories of Gagne, Ausubel, Piaget and Bruner as they relate to mathematics teaching.
- (3) describe some of the methods for teaching mathematics.
- (4) discuss some of the problems in mathematics teaching.
- (5) mention at least eight methods of teaching mathematics
- (6) describe at least six methods of teaching mathematics
- (7) write lesson note on some topics for at least six of the methods.

HOW TO STUDY

- 1 Read the unit very carefully.
2. Relate the content with your past experiences in the classroom..
3. Attempt all the activities
4. Ensure you practice some of the mathematics teaching skills in your micro-teaching exercise.

COGNITIVE THEORIES

Cognitive theories first appeared in the last century, but were usurped by behavioural theories earlier this century, only to re-emerge as the dominant force again. They are concerned with the things that happen inside our heads as we learn. They take the perspective that students actively process information and learning takes place through the efforts of the student as they organize, store and then find relationship between information, linking new to old knowledge, schema and scripts. Cognitive approaches emphasize how information is processed.

The three researchers Ausubel, Bruner and Gagne take different perspectives but each presents ideas that add to the discussion on how people learn. Ausubel's advanced organizer is a concept that considers the impact of prior learning. This differs from the behaviourists who do not consider the importance of this factor. Bruner's work on categorization or the forming of concepts provides a possible set of answers to how the learner derives information from the environment. Gagne looks at the events of learning and instruction as a series of phases, using the cognitive steps of coding, storing, retrieving and transferring information.

These three cognitive theorists, Jerome Bruner, David Ausubel and Robert Gagne have not adopted a developmental perspective. Although they have adopted quite different theoretical positions, they share the following features in common:

- they all put forward their ideas initially in the 1960s. At that time all three were established in their careers and recognized as authorities in their own right.
- All three attempted to define cognitive theories of instruction. The advent of these theories coincided with a period in which Western educators were, for the first time since the 1920s, seriously pausing to consider their educational policies; the depression and the Second World War had made such evaluations impossible for almost thirty years. Of equal importance was the fact that this period of questioning in the 1960s coincided with, periods of tremendous growth in scientific knowledge and expansion of, what was now in these Western countries, universal secondary education.

Bruner

The idea of Bruner, who advocated discovery learning, probably have had greater acceptance, at least in schools, than those of Ausubel or Gagne. In 1966, Bruner wrote "Toward a Theory of Instruction", in which he explained how his ideas might be translated into practice in the classroom. A further factor which contributed to the popularity of Bruner's ideas was that they were very much in tune with the mood of the times. His emphasis on discovery and 'hand on' learning was in accord with Piaget's ideas. Certainly the constructive nature of his theory appealed to teachers and many of his principles are still employed by practicing teachers.

Bruner argued that we should teach the 'structure' of subjects. He advocated the introduction of the real process of a particular discipline to students. For example, when learning history, students should become involved in genuine historical enquiry. This might involve examining a bridge, a building, or even a head stone in a cemetery, then using the information acquired to trace records of various kinds in order to answer the questions generated about the origins, purposes, and history of that structure, or the life of the person concerned.

The three stages in Bruner's theory of intellectual development are:

- Enactive where a person learns about the world through actions or objects.
- Iconic where learning occurs through using models and pictures.
- Symbolic which describes the capacity to think in abstract terms.

Bruner's underlying principle for teaching and learning is that a combination of concrete, pictorial then symbolic activities will lead to more effective learning. The progression is: start with a concrete experience then move to pictures and finally use symbolic representation. Is that path familiar to any of the readers? Are there similarities between Bruner's principle and the procedure suggested for teaching numeracy? Have you ever taught algebra using this procedure? It works!

Another aspect of Bruner's theory, which has been enthusiastically included in some teachers' classroom, is Discovery Learning. This is not an easy teaching strategy to employ.

Ausubel

Ausubel's writings have not attracted the popularity of Bruner's works. However because much of his theory has been developed from research in mainstream cognitive psychology, many of his ideas have survived as part of information processing theory. Ausubel's most notable contribution was the notion of the advance organiser. We can think of the advance organiser as simply a device or a mental learning aid to help us 'get a grip' on the new information. Put in more difficult language, according to Ausubel, the advance organiser is a means of preparing the learner's cognitive structure for the learning experience about to take place. It is a device to activate the relevant schema or conceptual pattern so that new information would be more readily 'subsumed' into the learner's existing cognitive structure or mental depiction!

The other major contribution which Ausubel has made, is his emphasis on the active nature of reception learning. The distinction between rote and meaningful learning is an important one, and too often we as educators fail to make reception learning as meaningful as possible. The need to require learners to be active by underlining, by completing missing words, by rewording sentences, or by giving additional examples, cannot be over emphasized in this context. Can you see a link with behaviourism here?

Gagne

Robert Gagne built upon behaviourist and cognitive theories to recommend approaches to instruction. Much of Gagne's early experience as an instructional psychologist was spent tackling practical problems of training Airforce personnel. He dealt particularly with problems in determining just what skills and knowledge are required for someone to be an effective performer at a given job. Once job requirements were identified, the task then becomes one of determining how those requirements might best be learned by a person in training for the job. He suggested that a task would be best learned by following a specific sequence of nine events:

1. gaining attention;
2. informing the learner of the objectives
3. stimulating recall of prerequisite learning;
4. presenting new material;
5. providing learning guidance;

6. eliciting performance
7. providing feedback about correctness
8. assessing performance; and
9. enhancing retention and recall.

In addition he proposed that learning is like a building process which utilises a hierarchy of skills that increase in complexity. He also identified five major categories of learning:

- i. verbal information
- ii. intellectual skills
- iii. cognitive strategies
- iv. motor skills
- v. attitudes

His notions of task analysis and the importance of the correct sequencing of instruction are followed by most mathematics teachers when designing their programs. Gagne's approach is really that of an instructional designer and, although his ideas have developed quite remarkably over the last quarter of a century, you can still glimpse the skeletons of the principles used when he was responsible for designing training systems for World War II pilots

Gagne's theory of learning hierarchies could be said to be a teaching theory, which is easy to apply in some circumstances, but is not easily applied in other circumstances.

Many of his ideas are readily transferable to computer-assisted instruction. Without doubt, at least some readers will be familiar with his ideas, even if not with Gagne himself as their advocate.

The concept of Gagne's knowledge hierarchy leads to the assumption that it is important to present all the necessary lower-level facts before proceeding to teach at higher levels. Related to this is the concept that people can reason with higher-level concepts if they have learned all of the prerequisites lower-level information. Gagne's ideas have received wide acceptance in the specialized training field although teachers have also accepted some of his principles.

ACTIVITY

- 1 Discuss the implication of the following psychological theories to the teaching and learning of mathematics:
 - a Gagne theory
 - b Ausubel theory
 - c Brunner theory

METHODS OF TEACHING MATHEMATICS

In the study of pedagogy of mathematics, the point of view is sometimes that of the manner in which the subject matter is arranged and developed; and at other times, of the manner in which it is presented to the students. To introduce this distinction in nomenclature, the former has sometimes been called **Method** and the latter **Mode**. In this usage, one would speak of the **Analytic Method**, but of the **Recitation mode**. The distinction is not always easy to make. Not all processes of instruction can be readily classified as relating distinctly and exclusively either to the sequence and interrelation of the subject matter or to the devices by which it is made clear to the student. Nevertheless, the broad distinction exists and even though the term “method” has been used to denote both phases indiscriminately it may help to keep the distinction more explicitly in mind to use the terms, at least loosely, in the sense cited.

Methods in Mathematics

As teaching methods in mathematics may be mentioned the synthetic, the analytic, the deductive, the inductive, the heuristic, the laboratory.

The characteristics of these methods will be indicated briefly in the sequels. They are not mutually exclusively: they shade into each other, and the classification of the treatment of subject topic or problem under one or another method is often difficult.

1. Synthetic and Analytic Methods

The Synthetic and Analytic methods are so familiar that their characteristics need only be recalled by a word.

The **synthetic** proceeds from the known to the unknown; the **analytic** traces out a part from unknown to the known.

The synthetic says, “Since A is true, it follows that B is true”. The analytic says, “To prove B is true, it is sufficient to prove that A is true”.

The synthetic “puts together” known truths and by the combination perceive a truth therefore unknown; the analytic “pull apart” the statement under question into simple statements whose truth or falsity is more easily determined.

Examples

The usual form of statements of proofs in textbooks of elementary geometry is a good example of the synthetic methods.

Beginning with known definitions and assumptions (axioms); each proof, each step, is deduced from what is known.

The problem is to find for what of values of x , if $x^2 + px = q$

The problem is solved if the same problem is solved for

$$x^2 + px + p^2 = q + p^2$$

Each step in an analytic march has its reason and its purpose.

In the synthetic method the steps follow more or less blindly, the truth of each is evident, but why this step should be taken rather than some other is a mystery, and the final result is often reached with a disagreeable shock “How did the author find this proof?” is frequently asked by students.

The reply is that in all probability he found it is an entirely different way from that in which he presented it to the world.

The great advantage of the analytic method is that if it connects with the known at any point, not matter where, its task is achieved; the synthetic method, on the other hand has only a single point

2) **Deductive and Inductive Methods**

A word will serve to recall the character of the **Deductive** and the **Inductive Methods**. The deductive method proceeds from the general to the particular; the inductive, from the particular to the general; A typical deductive syllogism is:

All men are mortal

Socrates is a man.

Therefore, Socrates will die.

A typical inductive inference is:

The sun has risen each past day

Of which we have any knowledge

Therefore the sun rises everyday.

The deductive type of inference is precisely what has been defined in the previous chapter as the mathematical type. It is the final form of all mathematical reasoning, but it does not follow that the reasoning, which leads to the result, is entirely or even in part, of this type. On the contrary, it is usually largely inductive. “This problem seems like such and such that I have met before, I solve them in a certain way. Therefore I can solve the present problem in the same way.”

- 3) **Discovery Method:** as the name sound is a very suitable method used in teaching mathematics, at primary and post-primary institutions where the interest of the student needs to be aroused. Whilst using this method in teaching, the teacher only serve as a helper while the student carry out their test.

It’s believed that in any attempt in which the teacher teaches through discovery and the learner learns through discovery, learning always remains longer.

Characteristics of Discovery Approach

- 1) Knowledge acquired lasts longer
- 2) There is intrinsic satisfaction

- 3) Broad future application
- 4) Increases intellectual potentialities
- 5) Encourage spirit of inquiring and exploration
- 6) Often occurs under democratic atmosphere
- 7) It answers the question, what, how and why of a particular learning
- 8) It is student-centred

Disadvantage

- 1) It's too-time consuming.
- 2) Less is covered within a stipulated time
- 3) May discourage the below-average student
- 4) Objectives are difficult to state in behavioral terms
- 5) Could be expensive

4) **Heuristic Method**

Heuristic approach to teaching is a sort of individualized type of student-centered learning. In heuristic methods (or sometimes called the genetic method) students are grouped or sometimes are given chance individually to discover facts on their own. Though it is discovery method in disguise, but here, no guidance is needed from the teacher. It is student-centred as against being teacher-centred.

As a method of teaching it has its own features. These are

- 1) Students develop much of the algorithm or at least some mathematics of their own (algorithm- set of steps or procedures for doing a calculation).
- 2) It involves active participation by the students.
- 3) It encourages both immediate and future transfer of learning.

Among its disadvantages as a method of instruction, we have

- i) It is fairly difficult (It is time consuming)
- ii) It discourages those with low I.Q.
- iii) It makes the position of the teacher redundant.

Note: Most of its disadvantages correspond to those of the discovery method.

5) **Analytic Method**

This method of teaching requires a certain level of understanding of concept (from the subject-often the student).

In addition, the students have some relevant knowledge, comprehensive and some fairly remote principles, to be able to solve problem. It is student-centred but sometimes the presence of a teacher is essential

It will be a difficult method to employ in primary and junior secondary institution, but in some cases it could be used in the senior most classes. Solving problems in mathematics using this method, involves a series of tests (which may appear awkward)

Mostly, question, under proof falls into this category, so that in most cases the student has to analyze its problem before arriving at a solution

In this way, the problem solver knows why each step is taken, i.e. he is the developer of algorithm

Again in a way, the problem solver is the creator of the solution

Synthetic Method: In the same way as in analytic method, synthetic method requires some certain amount of relevant knowledge and comprehension, before problem under it could be attempted.

In most cases, the teacher allows the students to abstract or symbolize where necessary before arriving at the solution.

The method is suitable for higher classes of secondary school or is best suitable in tertiary institution.

6) **Laboratory**

Mathematics laboratory is firstly a place where children can handle materials, perform mathematical experiments, play mathematical games, and become involved in other activities.

It is also a process/procedure of teaching and learning mathematics

The laboratory approach allows pupils to set up mathematics experiments for the purpose of discovering some mathematical principles, pattern or process. Mathematics laboratory also includes attitude. One of the purposes of the laboratory is to get the children to think for themselves, to ask questions and to look for patterns.,

In short, to develop an attitude of enquiry, children needs considerable experiences with the process.

To gain this kind of experience there should be a place rich in materials to which children have ready access. In other words, the three aspects of mathematical laboratory go together. The major aim of using a mathematics room is to let more able pupils have their understanding of mathematics sharpened by practical works and to make less able aware of the many areas of real life in which mathematics is important.

From the concrete experiences of pupils in mathematics laboratory, the less able pupils might gain some insight into mathematics.

Aspect of Mathematics Laboratory

- 1) Free exploration of facts
- 2) Directed exploration, i.e. exploration on a centred problem
- 3) Practical investigation.

Investigation can play a vital role in mathematics learning and understanding. The teacher sets the scene providing real materials, or a challenging problem when necessary. He observes what his pupils do with this. He asks questions, which will, and (help) them in their learning

The three essential stages in learning by investigation

- 1) **Free exploration**- a problem arises or teacher asks question, the children use materials to solve the problem
- 2) **Direct session**- Investigation of a particular idea which has developed during the first stage
- 3) **Practice session**- Group work is very valuable in this type of investigation especially in stage 1 and 2. Pupils discuss, interchange and develop ideas and achieve far more than they would when working individually

ACTIVITY

Do the following assignment.

1. For each of the methods discussed above, take a mathematics topic from primary syllabus to **SS 3** and illustrate how you can apply the method.

LEARNING AIDS IN THE TEACHING AND LEARNING OF MATHEMATICS

Teachers are always interested in looking for ways to improve their teaching and to help students understand mathematics. Research in England, Japan, China and the United States supports the idea that mathematics instruction and student mathematics understanding will be more effective if manipulative materials are used (Canny, 1984; Clements & Battista, 1990; Deines, 1960; Driscoll, 1981; Fennema, 1972, 1973; Skemp, 1987; Sugiyama, 1987; Suydam, 1984). Mathematics manipulative is defined as any material or object from the real world that children move around to show a mathematics concept.

Manipulative materials in teaching mathematics to students hold the promise that manipulatives will help students understand mathematics. At the same time, as with any 'cure', manipulatives hold potential for harm if they are used poorly. Manipulatives that are improperly used will convince students that two mathematical worlds exist – manipulative and symbolic. All mathematics comes from the real world. Then the real situation must be translated into the symbolism of mathematics for calculating. For example, putting three goats with five goats to get eight goats is the real world situation but on the mathematics level

we say $3+5 = 8$ (read three add five equals eight). These are not two different worlds but they are in the same world expressing the concepts in different ways.

What are manipulative materials? Manipulative materials are concrete models that involve mathematics concepts, appealing to several senses, that can be touched, felt and moved around by the students (not demonstrations of materials by the teacher). The term 'manipulative materials' raises one fundamental question, namely, 'just what are manipulative materials? In this context manipulative materials are objects or things that the pupil is "able to feel, touch, handle and move. They may be real objects which have social application in our everyday affairs, or they may be objects which are used to represent an idea'. Hence, not all teaching aids or instructional materials are manipulative materials. Suffice it to say here that manipulative materials appeal to several senses and are characterized by a physical involvement of pupils in an active learning situation. The manipulative materials should relate to the students' real world. For example, the use of an abacus is not something that is used in Nigerian daily life. Instead stones, eating utensils, tins, beans, oranges, mangoes, match sticks, etc. would be more appropriate.

Each student needs materials to manipulate independently. Demonstrations by the teacher or by one student are not sufficient. With students actively involved in manipulating materials, interest in mathematics will be aroused. Manipulative materials must be selected that are appropriate for the concept being developed and appropriate for the developmental level of the students. For example, one stick may be placed on a place value chart in the ones place. However, one stick should not be placed in the tens place. Instead a package of ten sticks bundled together with string or an elastic should be placed in the tens place. Students need to realize and conceptualize the idea of tenness. The same is true for the concept of the hundreds place; a bundle of 100 identical things should be used. As the student's concept of place value develops, then single sticks can be used for place value of numbers with greater value.

Good mathematics manipulative materials are durable, simplistic (easily manipulated), attractive (to create interest), and manageable. A systematic method should be developed for storage and distribution of the materials. Baskets or boxes are convenient for storage and distribution purposes.

Using manipulative materials in teaching mathematics will help students learn:

1. to relate real world situations to mathematics symbolism.
2. To work together cooperatively in solving problems.
3. To discuss mathematical ideals and concepts.
4. To verbalize their mathematics thinking
5. To make presentations in front of a large group.
6. That there are many different ways to solve problems
7. That mathematics problems can be symbolized in many different ways.
8. That they can solve mathematics problems outside of just following the teacher's directions.

If mathematics is taught using manipulative materials, then the methods of evaluating mathematical achievement must also change. Just calculating correct solutions to mathematics problems is not sufficient. Concept development and understandings should be valued more highly. Effective use of mathematics manipulative contributed to conceptualization and understanding. Evaluation of students' mathematics is changing from tests and testing to assessment. Assessment is much broader than testing or evaluation. For teachers to assess students' understanding of concepts, different techniques of evaluation will be needed. Teachers will receive more insight into students' mathematics understanding by::

1. Listening to students' talk about their mathematics thinking.
2. Observing students working individually and in cooperative groups
3. Asking 'why and how' questions rather than asking:
 - a. yes or no question
 - b. for results of calculating activities
 - c. For answers.
4. Having students write a solution to a problem rather than by only responding with correct or incorrect values.

Paper-and-pencil method of assessment limits the scope of student evaluation. Requiring students to defend their mathematical reasoning provides insight in the development of the students' thinking skills. Observation of students' functioning within a group will provide data for assessment. The teacher will move around the classroom observing how students are working and interacting.

To facilitate collecting assessment data, different types of questions will need to be asked by the teacher. The traditional questions which focus on calculating and correct answers will change to:

1. 'how and why' questions
2. Probing questions to stimulate the thinking process of the students.
3. Having students write responses to mathematical problems. This procedure helps in the following ways:
 - a. integrates writing with mathematics
 - b. inculcate in the students that numerical values are not sufficient for answer s to mathematics problems
 - c. presents an opportunity for reflection, which is conducive to students' cognitive development.
 - d. helps identify students having mathematical difficulties.
 - e. helps identify the conceptual level of development of the students.

Some examples of appropriate questions and responses for students might be:

1. How do you know that?

2. What would happen if ...?
3. Why do you suppose ...?
4. What makes you think your answer is correct?
5. How could you prove your answer is correct?
6. Could you express your answer in a different way?
7. What is another way to solve the problem?
8. How many different ways can you find to solve problems?
9. How can you convince the other members of your group that your way is the best method to solve the problem?

ACTIVITY

1. What are manipulative materials?
2. Why are manipulative materials necessary in the teaching and learning of mathematics?

WHY DO STUDENTS WITH LEARNING PROBLEMS STRUGGLE IN MATHEMATICS?

Research in the area of mathematics and students with learning problems is lacking in comparison to research in the areas of reading and language skills for these students. However, research can inform us about how to best help students with learning problems learn both mathematics concepts and mathematics skills (Mercer, Jordan, & Miller, 1996, Mercer, Lane, Jordan, Allsopp, & Eisele, 1996; Miller & Mercer, 1997). Students with learning problems struggle to learn mathematics for a variety of reasons. Two critical areas of research inform us why students with learning problems have difficulty learning mathematics. These two areas include the learning characteristics of the students with mathematics learning problems, and instructional issues that negatively impact the learning characteristics of these students.

Learning Characteristics of Students with Mathematics Learning Problems.

There are several research-based characteristics that negatively impact these students' ability to learn mathematics. The following learning characteristics represent substantial barriers to mathematics success for these students.

- **Learned Helplessness** is the result of students' continued failure with mathematics and the temptation we as educators have to 'get them through' a current set of problems without teaching them the underlying concept of the skill they are procedurally working. This results in both dependence on help from someone else and non-understanding. From these experiences, students learn, or more accurately, they are taught that they cannot learn mathematics.

- **Passive Learning:** Learned helplessness and metacognitive deficits (described below) produce a passive learning approach to mathematics for these students that prevent them from being ‘active’ learners. For example, a student may be confronted with the problem, 8×4 , he is not able to solve it. Although he may understand the rule of repeated addition (that $8 + 8 + 8 + 8 = 32$), he does not apply this knowledge to the ‘new’ situation. Due to his prior experiences with failure in mathematics, he is unwilling to take risks. Additionally, metacognitive deficits may prevent him from applying this strategy ‘automatically’.
- **Memory Problems:** Successful learners are able to retrieve from memory critical information when problem-solving. Students with memory problems often have difficulty doing this. They have a ‘faulty’ or inefficient memory retrieval mechanism. Difficulty with retrieving information from memory is especially problematic when these students are confronted with multi-step problem-solving situations. As they problem-solve, students often come to a point in the problem solving-process where they can’t retrieve from memory what they should do next.
- **Attention Problems:** Students with attention problems often ‘miss’ important information about solving; they have gaps in their knowledge base, which become barriers for accurate problem-solving. For example, students may miss the ‘subtract’ step in the ‘divide, multiply, subtract, bring down’ long division process.
- **Metacognitive Deficits:** Metacognition involves the ability to apply appropriate learning strategies, to evaluate their effectiveness, and to change strategies when current ones are not successful. Metacognitive deficits become more pronounced as students are expected to apply strategies and other information they have learned to new concepts and skills. The multiplication/repeated addition illustration under Passive Learning is one example of how metacognitive deficits affect mathematics learning.

Mathematics instructional issues that negatively impact the learning characteristics of students with learning problems

- **Spiral Curriculum:** Although the intent of a spiraling curriculum is good, the way it is often operationalised can be very detrimental to students with mathematics learning problems. Most notably, students with learning problems often do not get the chance to master a concept/skill before the class moves to the next concept/skill. When they revisit the concept/skill the following year, these students don’t have it in their skill repertoire because they never mastered it. This situation creates significant gaps in knowledge as they move through the elementary and secondary grades.
- **Teaching Understanding vs. Algorithm Driven Instruction –** The tendency for educators to embrace one way of teaching understanding without teaching algorithms places these students at risk of never developing the skills necessary for solving real life arithmetic situations and arithmetic required on standardized tests. Likewise, an emphasis on algorithms without teaching the conceptual understanding that underpins the algorithm creates procedural understanding without conceptual understanding.

Neither of these options bode well for students with learning problems. Students with learning problems often receive a heavy emphasis on algorithm instruction because that's what we 'see' them struggle with most. And, for many of us, that is what we experienced mathematics to be as students. Because of the various learning characteristics that negatively impact these students, they may always have difficulty performing algorithms efficiently. However, without teaching them conceptual understanding while we are teaching algorithms, they will not understand foundational mathematics concepts either. Teaching one and not the other places these students in a precarious position. They become masters of neither of the important skills.

- Cyclical Reforms – Like reading, mathematics undergoes periodic 'reforms' which are cyclical in nature. While reading instruction moves between phonic-driven instruction and whole-language instruction, mathematics moves between algorithm driven instruction and teaching conceptual understanding. While successful learners can manage the transition these cyclical reforms make during their school career, students with learning problems do not manage such transitions well, causing significant gaps in conceptual and skill acquisition.
- **Application of effective teaching practices for students with learning problems:** Students with mathematics learning problems need research-supported instructions in order to learn mathematics. Moreover, these instructions must meet their unique learning needs. The good news is we know what works for these students. The bad news is that such practices are poorly integrated in many mathematics textbooks/series and therefore are not implemented systematically in classrooms. The result for these students is instructions that do not meet their learning needs and therefore limit success.

ACTIVITY

- 1 Enumerate some barriers to mathematics success.
- 2 Discuss instructional issues that negatively impact the characteristics of students with learning problems.

SUMMARY

In this unit you have learnt that:

- It is certain that good results cannot be achieved if the school child has no enthusiasm for his work; if there is no strain on his part, no matter how much effort the teacher may put into explanation, individual consultations and supplementary lesson, little would be achieved.
- The teacher's work goes for naught unless the pupils goes deeply into the essence of the matter and tries to comprehend the subject himself.

- Loss of interest in learning at some stage gives rise to indifference and apathy; indifference gives rise to laziness, and laziness gives rise to idleness and loss of ability. That is why it is important to rethink the methods of teaching mathematics, so as to make its study interesting.
- Mathematics teachers should learn to direct their attention towards the facilitation of students' understanding and conceptualization rather than drill and practice of rote procedures.
- The use of manipulative materials in mathematics classrooms supports this attempt.
- Incorporating the use of manipulative materials with an emphasis upon the thought process of students provides an opportunity for the teacher to assess and meet the needs of primary school students as they construct personal mathematical knowledge.
- Barriers to mathematics success should be removed.
- Instructional issues that negatively impact students' learning should be redressed.

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UNIT 4: EDUCATIONAL OBJECTIVES, LESSON PLANNING, CONTINUOUS ASSESSMENT AND TEST CONSTRUCTION

INTRODUCTION

The most important aspect of planning for teaching or testing is the determination of the general and specific objectives of the course. Teaching or testing in the absence of objectives is analogous to beginning a trip before deciding where you want to go. Proper planning is important to the success of any major endeavor. It is therefore necessary to consider the formulation of objectives. On no account should a teacher prepare a lesson plan without describing the objectives to be achieved.

OBJECTIVES

By the end of the unit, you should be able to

- (i) distinguish between general objectives and specific objectives
- (ii) define behavioral objectives
- (iii) discuss Taxonomy of Educational Objectives
- (iv) define what a table of specification is
- (v) draw a table of specification for a topic and several topics
- (vi) state the advantages of table of specification

HOW TO STUDY THIS UNIT

1. Read the unit very carefully.
2. Relate the content with your past experiences in the classroom..
3. Attempt all the activities
4. Ensure you practice some of the mathematics teaching skills in your micro-teaching exercise.

GENERAL AND SPECIFIC OBJECTIVES

Educational objectives may be placed on a continuum from the most general to the most specific. When objectives are properly used the specific ones are classified under the more general ones, which they promote. Objectives are quite interrelated. One specific objective may promote several different general objectives.

The teacher however, should have a clear idea of the particular objectives he is trying to further. He should know what school-wide objectives his course is designed to promote, what course objectives various units of the course are designed to promote, and what unit objectives are being advanced by daily activities.

General Objectives

The most general objectives are the aims of the total educational effort. These are the purposes for which societies establish and maintain schools. General objectives in a particular subject area, say mathematics, are related to why we need to study mathematics. The general objectives of mathematics education at various levels of education reflect the values the society expected from someone who has studied a particular mathematics curriculum.

Special Objectives

Specific objectives are different from general objectives in that they usually contain only one kind of knowledge, one skill or one attitude, whereas general objectives may involve several.

Behavioural Objective

In order to build a test that accurately measures the achievement of what you want students to learn, your objectives must be stated in terms of specific student behaviour that you can observe. You should note it is the student who achieves, and it is the student who will exhibit his achievement by taking a test. The objectives you will use to build your test must, therefore, be stated in terms of a particular kind of behaviour that you will ask the student to exhibit in a testing situation. In stating behavioural objectives, you need to use action verbs. Never use passive verbs like 'know', 'understand'. Behavioural objectives are the real measurable expectation or behaviour you expect at the end of a particular learning experience. We may state behavioural objectives thus:

At the end of the lesson, students should be able to

- (i) Add up sum of two digits numbers
- (ii) Solve quadratic equation using completing the square method
- (iii) Draw a graph of quadratic equation
- (iv) Solve word problems involving simultaneous equation
- (v) State the Pythagoras theorem.

The words add, solve, draw state are measurable attributes, which can be seen to have been performed.

NOTE:

Never use the expression like; 'to know', 'to understand' in writing behavioural objective because 'know' and 'understand' are not active verbs.

THE TAXONOMY OF EDUCATIONAL OBJECTIVE

The taxonomy of educational objectives otherwise referred to as behavioural objectives was based on study conducted by Bloom and his associates which started in 1948 and the first handbook was published in 1956.

The taxonomy group first classified behaviour into three major domains cognitive, affective, and psychomotor. The cognitive domain covered all those activities generally thought of as mental functions, such as knowing, understanding and analyzing.

The affective domain includes the emotional or feeling aspect of an individual, such things as attitudes and beliefs. The psychomotor domain included, generally, physical activities. Objectives in the psychomotor domain would be expected not only in physical education courses but also in such courses as writing in the primary grades and typing in secondary school.

The cognitive domain is divided into six major level, from the simplest behaviour, to the most complex. The levels are not mutually exclusive. In many cases the more complex behaviours are dependent on prior attainment of the simpler ones. The six levels are: knowledge, comprehension, application, analysis, synthesis and evaluation.

Knowledge: The behaviour involved here is primarily that of remembering and being able to recall information. No understanding is implied at this level. For example, define Pythagoras theorem, what is the sum of the angles of a triangle, etc.

Comprehension: This is usually what the teacher calls understanding. It basically involves getting the meaning of something. Generally speaking, isolated facts are not comprehended-only memorized relations among facts, implications of factual information, generalizations, procedures, concepts and ideas are the kinds of things that are comprehended.

Application: The behaviour implied here is the student's ability to use in a concrete situation the information, procedures, and ideas, which he has acquired and comprehended.

Analysis: This involves an investigation and determination of the structure of something, the method or procedure of a communication rather than the messages communicated. We may analyze something by determining its structural components, by noting their interrelationships, and or by determining its type. Objectives which involve the behaviour of analysis are found in all subjects and at all levels of education.

Synthesis: This is what some call invention or creative thinking. Synthesis is the creation of something new, new at least, for the person creating it. It might involve writing a story, discovering a procedure, formulating a new hypothesis, or drawing a generalization.

Evaluation: Once something has been created it may be evaluated. Evaluation is considered to be the most complex behaviour in the cognitive domain. It involves most if not all of the other cognitive behaviours. For example, before something can be evaluated one must have sufficient knowledge of it, must comprehend what he knows about it, and must be able to apply the procedure of analysis and synthesis. Things are evaluated in terms of criteria established for their evaluation.

Lesson Presentation

It is important to plan well ahead of time so that the teacher may be adequately informed on the possible problems that may arise and to source for necessary teaching materials before the lessons are taught.

Kinds of Lesson Plan

There are usually two kinds of lesson plan, namely: Unit Plan and Daily Plan.

Definition: A unit plan covers the work on a particular topic for two or three days, a week, or more than a week, on the school's scheme of work. The work to be done in a particular mathematics topic is always specified for one, two or more weeks.

The purpose of a unit plan is to arrange for the teaching of an entire topic no matter how long it takes to teach that topic. It is a better substitute for the weekly plan for various reasons. The unit plan spells out its objective(s), the content to be taught, the method of approach and the content of the final assessment, which will be used to evaluate the achievement of the pupils at the end of that unit. The major headlines for a unit plan are as follows:

- (1) Topic
- (2) Objectives
- (3) Teaching materials
- (4) Basic knowledge/Pre-requisite skill/Pre-test
- (5) Content (The number of lessons to be taught and the content of each lesson should be stated clearly).
- (6) Method
- (7) Final assessment
- (8) References

Daily Plan:

The Daily Plan is prepared for a day's lesson at a time taking into consideration the achievement of the pupils in previous lessons and the detailed content provided in the plan.

All daily plans are based on the appropriate unit plan. The format of the daily plan is similar to that of the unit plan, except that, it is more detailed. The format is as follows:

1. Objective(s)
2. Teaching Materials
3. Basic knowledge/skill
4. Content (You may sometimes give examples for clarity)
5. Method/procedure
6. Assessment/Assignment
7. Teacher's assessment of lesson.

It is useful for the teacher to assess the performance of the pupils as well as of his/her teaching. Feedback from the teacher would help to improve subsequent instruction. Teachers should therefore try to write comments on their lesson in item 7 of their daily plan.

We now discuss a specific example. We write unit and daily plans for teaching everyday statistics in primary four.

Unit Plan

Class: Pry IV

Topic: Everyday statistics

1. **Objectives:** Pupils will be able to
 - i. Read pictogram and show information in pictogram using vertical and horizontal arrangements.
 - ii. Identify the most common value from a pictogram.
2. **Teaching Materials:** Matchboxes, pebbles, oranges, groundnuts, chalk, and ruler blackboard.
3. **Basic Knowledge:** Counting numbers from 1 to 100
4. **Content:** Three lessons: Pictogram, Mode
- Lesson 1: Reading pictogram and showing information in pictogram using vertical arrangements.
- Lesson 2: Horizontal Arrangement
- Lesson 3: Identification of the most common object.
5. **Method:** The listed materials will be brought to the class. We start with oranges (20 oranges). Select 4 pupils. Divide the oranges among them, first: 3 second 4, third 6, fourth: 7. the four pupils stand in front of the class and the oranges are vertically in front of them. The information is recorded on the board. The procedure is repeated for groundnuts and pebbles.
6. **Assessment:** Pictogram of some objects is arranged vertically on the board. Pupils asked to tell number of objects of each listed name on the board.
7. **References:** Primary mathematics for Nigerian Schools NERDC Book 4.

Daily Lesson Plan

Class: Primary IV

Topic: Everyday Statistics

Objectives: At the end of the lesson pupils shall be able to read pictogram and show in information in pictogram using vertical arrangement.

Teaching Materials: 10 match boxes, 20 oranges, 30 groundnuts, chalk, ruler and blackboard.

Basic knowledge: Counting numbers from 1 to 100

Content: forming pictogram, showing pictogram vertically. Reading pictogram.

Method: We start with 10 matchboxes, select 3 pupils and identify the names of the pupils.

We divide the boxes as 2, 3, 5 say Taye 2, John 3 and Mary 5. Arrange the boxes in front of them vertically. Write the information on the board using the names. Repeat for oranges and groundnuts.

Assessment: A pictogram of certain number of milk tins is drawn on the board against 4 names. Pupils are to say how many tins each named person has.

Assignment: Pupils are asked to bring 5 pebbles to class the next day.

Teacher's Assessment of Lesson: The lesson is good, but it would be better to use more objects.

ACTIVITY

- 1 Write behavioural objectives for the following topics:
 - (a) Recognition of numbers (for Nursery 1 Pupils)
 - (b) Addition of two digits numbers (with carrying)
 - (c) Ratio
 - (d) Variation
 - (e) Fractions, decimal and percentage
- 2 Take WAEC or NECO past objective questions in mathematics and categorize all the questions either as knowledge, comprehension, application, analysis, synthesis or evaluation.
- 3 Write a lesson note to teach the topic. Simultaneous Equation to JSS 3 students.

CONTINUOUS ASSESSMENT AND EVALUATION

Evaluation of educational programme has for quite a long time been accorded a significant process, through which the strengths and the weaknesses of teaching strategies could be determined. It is therefore necessary for the teachers of the various school subjects to know how to effectively use this tool.

What is Continuous Assessment?

Continuous assessment is a systematic use of varied and reliable multiple assessment tools at regular intervals to determine the performance and ability of the learner in the three domains of behavior with the aim of getting his truest picture and helping develop fully his potentials, Emeke (1996).

It is also the systematic and objective method of determining the extent of learners' performance in the expected changes in his behavior, Onasanya (1991).

It should take effect from the first day of the learner in the school to the end of his course of study. It is usually conducted in a continuous and progressive manner and it provides a useful accumulation of all kinds of information so derived to guide and shape the learner in his day-to-day educational endeavors.

For such method to be effective, the phase or activities if contains should be implemented in a systematic fashion to ensure uniformity and comparability. The teachers are to know the following and get their students informed about it:

- i. The number of assessments scheduled for the term
- ii. Date of each assessment
- iii. Topic or modules on which assessment is to be based
- iv. The objectives domains to be assessed each time
- v. Types of instruments to be used.

Characteristics of Continuous Assessment

1. It is systematic
2. It is comprehensive
3. It is cumulative
4. It is guidance oriented
5. It should have predetermined ulterior
6. It should have good monitoring system

The Need for Continuous Assessment

Advantages

Teachers Part	Continuous assessment makes teachers more participating in the final decision making of the student's learning.
Preparation	It also makes the teacher and the students to prepare better for the teaching and learning.
Examination Malpractice -	It reduces examination mal practices in the sense that some percentage of the whole total has been dropped for continuous assessment and the part is not only paper and pencil test only. It includes the students' behavior, attitudes, and skills knowledge test.
Reading / Study Habit -	It develops the student better for the whole programme because when a student is preparing for continuous assessment, he goes through stage by stage all the requirements of the course or programme.
Domains	It allows the non-cognitive domains to be tested and the record kept of the student is not only on the cognitive aspect.

Places of Data Collection In Continuous Assessment

Entry Phase - This is when the students have just entered into a programme. In primary school there is only one phase i.e. Primary I.

In secondary school we have 2 phases i.e. JSSI and SSI – Data collected at this phase is used for identifying interest, socio-economic background, motivation for learning, learning disorder or disability, needs, status and intellectual ability of each student.

Passage Phase: Data collected here takes into account the cognitive development, affective, psychomotor behavior, learning experiences, the use of materials, supportive and intervention strategy of each student. Assessment here provides for monitoring the students' development and progress.

Terminal Phase: Data collected or assessment here recognizes ability, aptitude, attitude, interest and manipulative skills for the decision on the kind of secondary school, types of senior school or even tertiary institution for the students.

Record Keeping:

Progress Record Card – This is the most appropriate or important tool for school record keeping. The format will contain:

- i. Personal information about students
- ii. Weekly / Periodic report of academic achievement
- iii. Report of summary of academic performance per term
- iv. Affective and psychomotor term report
- v. Terminal examination
- vi. Yearly summary of progress.

Problems of Continuous Assessment

Some factors hinder the successful implementation of continuous assessment.

- I. **Assessing non-Cognitive Domains** - Teachers in our schools find it difficult to assess in the non-cognitive domains. This is because instruments for gathering data in the domains are not available in our schools. The problem however, is that of assembling and disseminating the instruments to the final users in schools.
- II. **Misconception:** Teachers and students in the educational system regard continuous assessment as continuous testing at cognitive level only.
- III. **Use of Instrument:** Course offered in Teacher Education Programs do not equip teachers with the techniques of continuous assessment. This is because the concept of continuous assessment is new in the educational system.

A large number of our school teachers in the secondary school are not trained about continuous assessment. Therefore the teachers are not skilled in the techniques of continuous assessment. A faulty use of instruments affects the standards within school and across schools.

RECOMMENDATIONS

1. To overcome these problems, intensive workshops to educate practicing teachers are necessary.
2. Efforts should be made to collect instruments hidden in our universities and libraries and make them available to teachers.
3. News letters and periodicals published in the ministry of education which focus on continuous assessment should be distributed to schools.

ACTIVITY

1. What are the characteristics of continuous assessment?
2. Why do we need continuous assessment?
3. What are the problems of continuous assessment?

TEST CONSTRUCTION

Test construction is part of teachers' duty. Teacher-made tests are essential aspects of teaching/learning process. One of the most efficient ways to utilize well-written objectives in test construction is by means of a table of specification. A table of specifications is essentially a two-way grid, with the content outlined along the vertical axis and the behaviours the student is supposed to accomplish along the horizontal axis.

Table of Specification

A table of specification is simply a convenient way of placing the content outline of a unit in relation to the behaviours to be promoted with that content. Since it contains the unit outline and objectives, it should be constructed at the very beginning of the unit plans.

The teacher uses the table of specifications to guide his teaching and measurement procedures. This is one way to insure that his testing parallels his teaching. Each item on each test given in connection with the unit may be related according to the content area and behavioural objectives which the item measures.

Table of specification based on Blooms Taxonomy of Educational Objectives

	Behavioural objectives with percentage emphasis						
	Knowledge	Comprehension	Application	Synthesis	Evaluation		
Content	28%	20%	16%	16%	10%	10%	
Simple Equation	4	2	2	3	1	1	13
Simultaneous Equation	3	3	2	2	2	1	13
Quadratic Equation	3	3	2	2	2	2	14
Surds	4	2	2	1	--	1	10
	14	10	8	8	5	5	50

Suppose you want to construct 50 objectives tests on four (4) content areas, as indicated in the table above.

The table of specification ensures that you distribute the questions spanning all objectives. In the above table the columns shows the six objectives to be measured while the rows represents the content areas to be covered in the test.

The table specification above covers four content areas. If you examine the table you will note that they differ in the amount of emphasis placed on each domain and each cognitive area. One or more areas may even be omitted in a table of specification. The table should have only those objectives that the teacher proposes to teach in that unit

Advantages of Table of Specifications

If a test is to be valid, it should measure what the teacher attempts to teach. The major advantage of a table of specification is that it enables the teacher to build content validity into his test. The table of specifications defines the universe of content and behaviours from which test exercises may be drawn. If test exercises are classified in table of specifications as they are written, then the teacher can easily see if he is drawing from the universe, which has been defined. He can also see if he is neglecting a significant objective or overemphasizing a minor one.

Another advantage of a table of specifications is that ii enables the teacher to have a clear perspective of a unit of work and the specific behavioural change he hopes to bring about through it. This more precise picture enables him to do a more efficient job of both teaching and testing.

A third advantage of a table of specifications is its diagnostic values for both the teacher and the learners. If the test items have been classified in a table of specifications, the learners can

determine the particular content areas and behaviours in which they are having difficulty. In the same way the teacher can check on his own effectiveness by noting areas and objectives in which substantial numbers of learners in a class are having difficulty.

The table of specification should be a major time-consuming undertaking just before a large end-of-unit examination. They should be constructed during the planning stage of the unit. The teacher can classify and make validity checks on the questions he asks in daily and weekly tests as the class progresses through the unit.

ACTIVITY

1. Develop a complete table of specifications for a unit in your teaching field
2. Take any WAEC OR NECO past mathematics objective questions paper. Classify the questions into six categories according to Bloom's taxonomy. Then form a table of specification for the past question paper chosen

SUMMARY

You have learnt in this unit:

- What a table of specification is?
- How to construct a table of specification
- The advantages of having a table of specification.
- How to construct a table of specification.
- Different kinds of objectives.
- How to write behavioural objectives.
- Bloom's Taxonomy of Educational Objectives.

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